Application of the Extension of the Category of Relations (CoR) to UAV Swarm for Search-and-Rescue Mission

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Keywords: UAV Swarms; Category of Relations (CoR); Multi-agent Systems; Search-and-Rescue; Optimization; Resource Management; Decentralized Systems

1 Introduction

UAV swarms deployed for search-and-rescue missions operate under complex, dynamic conditions. These missions require real-time, decentralized decision-making, and resource management. Traditional UAV swarm design approaches often fall short in addressing decentralized autonomy and adaptability in such unpredictable environments. The *Category of Relations (CoR)* framework, introduced in (Dickerson and Wilkinson, 2024), provides a formal and rigorous framework for structuring, analyzing, and designing complex systems. It extends traditional engineering methods by introducing formalized relationships, functors, and logical constraints, making system architecture more precise and mathematically grounded.

In this paper, we apply CoR to model and optimize UAV swarm design in a search-and-rescue mission, focusing on how UAVs interact with the environment and each other under constraints such as limited energy and communication bandwidth.

2 Application of CoR to UAV Swarm Design

Introduction to the Case Study: The search-and-rescue mission involves deploying a swarm of UAVs in a hazardous and unpredictable environment, such as after a natural disaster or in rough terrain. The UAV swarm operates as a decentralized multi-agent system, where each UAV makes local decisions based on its sensory data while collaborating with other UAVs to achieve the mission objectives. In this context, the mission's success relies on the UAVs' ability to communicate efficiently, manage resources such as energy and bandwidth, and adapt to changing conditions.

The CoR framework is applied here to formalize the relationships between system components (UAVs), resources, mission objectives, and environmental factors—allowing for structured and systematic analysis of the swarm's behavior and performance. By modeling the components as objects within CoR, we gain insight into how UAVs interact with the environment, consume resources, and collectively contribute to the mission's success.

System Components and Relations in CoR: The first step in applying CoR is to define the system's components (UAVs, resources, environment, and mission objectives) as objects:

A: The UAV swarm, where each UAV is responsible for data collection, navigation, and communication.

B: The environment, which includes obstacles, terrain, and potential victims to be rescued.

C: The resources available to the UAVs, including energy and communication bandwidth.

D: The mission objectives, such as area coverage and victim detection.

In CoR, each of these obects is a set containing the data associated to the object. The next step is to define the relationships (morphisms) between these objects. A morphism $R : A \to B$ in CoR is a subset R of $A \times B$. These morphisms capture how the UAVs interact with the environment, use resources, and contribute to the mission's success:

R_1 : $A \to B$: UAVs interacting with the environment (e.g., sensing and covering terrain).

R_2 : $C \to A$: UAVs consuming energy

R_3 : $C \to A$: UAVs consuming communication bandwidth.

R_4 : $A \to D$: UAVs achieving mission objectives (e.g., area coverage and victim detection).

Composition of Relations in CoR: One of the key features of CoR is the ability to compose relations to model how different system components interact through multiple layers. For example, the composition $R_3 \circ R_2$ represents how the resources consumed by UAVs (such as energy and bandwidth) impact their ability to achieve mission objectives (such as covering the search area or locating victims):

$$R_4 \circ R_2 = \{(c, d) \mid \exists a \in A, (c, a) \in R_2 \text{ and } (a, d) \in R_4\}.$$

This composition illustrates how resource limitations (e.g., energy depletion) can affect the swarm's ability to accomplish its objectives.

Optimization in CoR: Using CoR, we can define optimization objectives and constraints for the UAV swarm design. The objectives include minimizing the total enery consumption E_{total}, maximizing the covered area A_{total}, and minimizing communication bandwidth usage C_{total}. Constraints include ensuring that each UAV's energy consumption does not exceed its capacity $E_{\max}$, and that the total coverage area meets the mission's requirements A_{required}. Within the CoR framework, this yields the following optimization problem:

$$\begin{cases} \text{Minimize} \quad E_{\text{total}} = \sum_{i=1}^{N} R_2(c, a_i) \\ \text{Maximize} \quad A_{\text{total}} = \sum_{i=1}^{N} R_1(a_i, b) \\ \text{Minimize} \quad C_{\text{total}} = \sum_{i=1}^{N} R_3(c, a_i) \end{cases} \qquad \text{subject to constraints:} \qquad \begin{cases} \forall 1 \leq i \leq N,\ E_i \leq E_{\max} \\ A_{\text{total}} \geq A_{\text{required}} \end{cases}$$

By formulating the optimization problem within the CoR framework, we can systematically balance these objectives and constraints, ensuring that the UAV swarm operates efficiently under the given resource limitations.

3 Discussion

The CoR framework formalizes the interactions between UAVs, resources, and mission objectives, providing a structured approach to optimize UAV swarm performance. By modeling trade-offs between objectives like energy consumption and coverage, CoR enables designers to better manage resource usage in dynamic environments.

CoR's ability to model and optimize competing objectives is particularly useful in real-time scenarios. The use of relational homomorphisms ensures consistency across system layers, aligning resource allocation with mission goals.

Despite these advantages, CoR's effectiveness depends on accurate environmental data, and inaccuracies could result in suboptimal decisions. Future work could address this by incorporating probabilistic models to manage uncertainty.

As UAV technologies evolve, CoR offers a flexible, scalable alternative to centralized control, making it a valuable tool for complex scenarios such as disaster response.

The application of the Category of Relations (CoR) to UAV swarm design has demonstrated its effectiveness in formalizing and optimizing complex, multi-agent systems. By representing UAVs, resources, and mission objectives as objects and their interactions as morphisms, CoR enables a systematic approach to analyzing and optimizing UAV swarm performance.

CoR's ability to model trade-offs between conflicting objectives, such as energy consumption and coverage, makes it a valuable tool in designing UAV swarms that can operate efficiently in dynamic environments. Although there are challenges related to handling uncertainty and applying CoR to heterogeneous UAV swarms, future extensions of the framework could address these limitations and enhance its applicability.

References

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